



MATHEMATICS
HIGHER LEVEL PAPER 1

Wednesday 3 May 2000 (afternoon)

2 hours

Name

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Number

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INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. (a) Sketch the graph of $f(x) = \sin 3x + \sin 6x$, $0 \leq x \leq 2\pi$.
- (b) Write down the exact period of the function f .

Working:

Answer:

(b) _____

2. The transformation T_1 is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ and the transformation T_2 is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

- (a) Calculate the matrix $(T_1 T_2)^{-1}$.
- (b) Describe the transformation represented by the matrix $(T_1 T_2)^{-1}$.

Working:

Answers:

- (a) _____
- (b) _____

3. Let $z_1 = a \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$ and $z_2 = b \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$.

Express $\left(\frac{z_1}{z_2} \right)^3$ in the form $z = x + yi$.

Working:

Answer:

4. A sample of 70 batteries was tested to see how long they last. The results were:

Time (hours)	Number of batteries (frequency)
$0 \leq t < 10$	2
$10 \leq t < 20$	4
$20 \leq t < 30$	8
$30 \leq t < 40$	9
$40 \leq t < 50$	12
$50 \leq t < 60$	13
$60 \leq t < 70$	8
$70 \leq t < 80$	7
$80 \leq t < 90$	6
$90 \leq t \leq 100$	1
Total	70

Find

- (a) the sample standard deviation;
- (b) an unbiased estimate of the standard deviation of the population from which this sample is taken.

Working:

Answers:

- (a) _____
- (b) _____

5. Find the coefficient of x^7 in the expansion of $(2 + 3x)^{10}$, giving your answer as a whole number.

Working:

Answer:

6. The system of equations represented by the following matrix equation has an infinite number of solutions.

$$\begin{pmatrix} 2 & -1 & -9 \\ 1 & 2 & 3 \\ 2 & 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ k \end{pmatrix}$$

Find the value of k .

Working:

Answer:

7. In a game a player rolls a biased tetrahedral (four-faced) die. The probability of each possible score is shown below.

Score	1	2	3	4
Probability	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	x

Find the probability of a total score of six after two rolls.

Working:

Answer:

8. Find a vector that is normal to the plane containing the lines L_1 and L_2 , whose equations are:

$$L_1: \mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

$$L_2: \mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu (\mathbf{j} + 3\mathbf{k})$$

Working:

Answer:

9. The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 - 2n$. Find the n th term u_n .

Working:

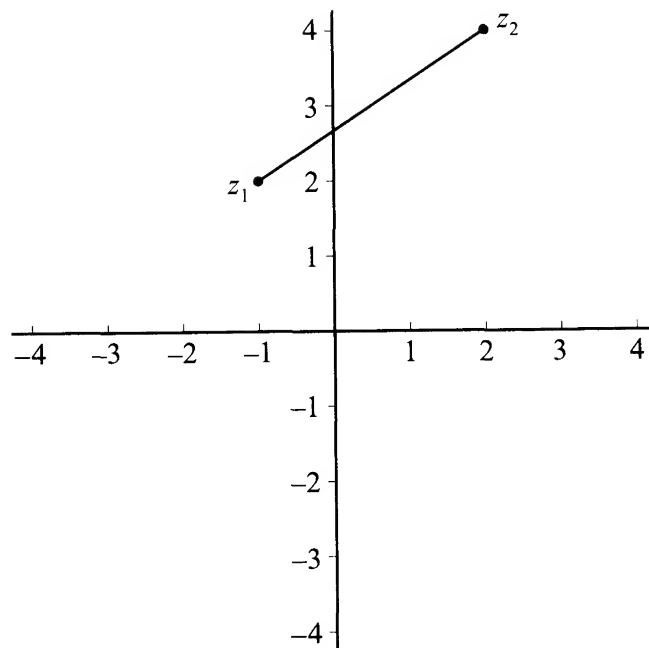
Answer:

10. The plane $6x - 2y + z = 11$ contains the line $x - 1 = \frac{y + 1}{2} = \frac{z - 3}{l}$. Find l .

Working:

Answer:

11. The points $z_1 = -1 + 2i$ and $z_2 = 2 + 4i$ and the line segment $[z_1 z_2]$ are shown in the complex plane below.



Let $z_3 = -i z_1$ and $z_4 = -i z_2$.

- (a) Plot z_3 and z_4 on the complex plane and draw the line segment $[z_3 z_4]$.
- (b) Write down the transformation that maps the line segment $[z_1 z_2]$ onto the line segment $[z_3 z_4]$.

Working:

Answer:

(b) _____

12. The probability distribution of a discrete random variable X is given by

$$P(X = x) = k \left(\frac{2}{3} \right)^x, \text{ for } x = 0, 1, 2, \dots$$

Find the value of k .

Working:

Answer:

13. The velocity, v , of an object, at a time t , is given by $v = ke^{-\frac{t}{2}}$, where t is in seconds and v is in m s^{-1} . Find the distance travelled between $t = 0$ and $t = a$.

Working:

Answer:

14. Mr Blue, Mr Black, Mr Green, Mrs White, Mrs Yellow and Mrs Red sit around a circular table for a meeting. Mr Black and Mrs White must not sit together.

Calculate the number of different ways these six people can sit at the table without Mr Black and Mrs White sitting together.

Working:

Answer:

15. Find the coordinates of the point which is nearest to the origin on the line

$$L: x = 1 - \lambda, y = 2 - 3\lambda, z = 2.$$

Working:

Answer:

16. Given that $x > 0$, find the solution of the following system of equations:

$$\frac{8x^3}{y} = 3$$

$$xy - y = x^2 + \frac{9}{4}$$

Working:

Answers:

17. A rectangle is drawn so that its lower vertices are on the x -axis and its upper vertices are on the curve $y = \sin x$, where $0 \leq x \leq \pi$.

- (a) Write down an expression for the area of the rectangle.
- (b) Find the maximum area of the rectangle.

Working:

Answers:

(a) _____

(b) _____

18. Find the values of $a > 0$, such that $\int_a^{a^2} \frac{1}{1+x^2} dx = 0.22$.

Working:

Answers:

19. Let $f: x \mapsto e^{\sin x}$.

(a) Find $f'(x)$.

There is a point of inflexion on the graph of f , for $0 < x < 1$.

(b) Write down, but do not solve, an equation in terms of x , that would allow you to find the value of x at this point of inflexion.

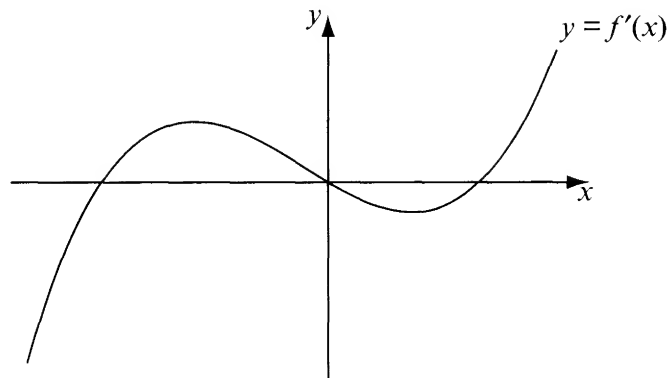
Working:

Answers:

(a) _____

(b) _____

20. The diagram shows the graph of $y = f'(x)$.



Indicate, and label clearly, **on the graph**

- (a) the points where $y = f(x)$ has minimum points;
- (b) the points where $y = f(x)$ has maximum points;
- (c) the points where $y = f(x)$ has points of inflexion.

Working: